SoK: Efficient Design and Implementation of Polynomial Hash Functions over Prime Fields

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Outline

Δ-Universal Hash in Practice

- **Definition:** Given $z \in \mathcal{T}$ and $M \neq M' \in \mathcal{M}$,

$$\Pr_{r \leftarrow \$ \mathcal{R}}\left[H_r(M) - H_r(M') = z\right] \leq \epsilon(M, M')\,.$$

- **Various practical applications:**
  - Data Structures: hash tables [CW79].
  - Message Authentication Codes: UMAC, Badger, Poly1305-AES, GMAC [ISO/IEC 9797-3].
  - AEAD: AES-GCM, ChaCha20-Poly1305 [RFC 8446].

The Adoption of ChaCha20-Poly1305 (ChaChaPoly)

2005,08 -Poly1305 and ChaCha20 designed separately by Bernstein.

2013 -First ChaChaPoly IETF draft, supported in chrome and OpenSSH.

2015 -ChaChaPoly specified for IETF protocols in [RFC 7539].

2016 -ChaChaPoly proposed standard for **TLS** in [RFC 7905].
-Default choice in OpenSSH and WireGuard.

2019 -Default choice in OTRv4 and the Bitcoin Lightning Network.

Key Points:

- Good performance across all architectures without needing specific hardware support.
- Alternative and backup AEAD scheme to AES-GCM.
- Fast adoption even with the predominance of AES-GCM.
- Conservative and simple design, focused on performance with standard AEAD security.

Poly1305 [Ber05]

For $M = M_1 \| \cdots \| M_n$,

$$\text{Poly1305}(r, M) = (c_1 x^n + c_2 x^{n-1} + \cdots + c_n x^1 \mod 2^{130} - 5) \mod 2^{128},$$

where $c_i = M_i \| 1$ and $x = \text{clamp}(r, 22)$.

Limitations:

- Clamping introduced for fast implementations using FPUs (Floating-Point Units).
  $\rightarrow$ Almost all implementations of Poly1305 use integer ALUs (Arithmetic Logic Units).
  $\rightarrow$ Provides only ≈103 bits of security with a 128-bit key and tag.
- Tailored for 32-bit architectures.
- Limited security of ChaChaPoly in the multi-user setting due to Poly1305 [DGGP21].

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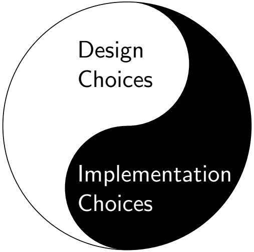
Given today's advancements and applications, would we still converge to this same design?

Systematization of Knowledge (SoK)

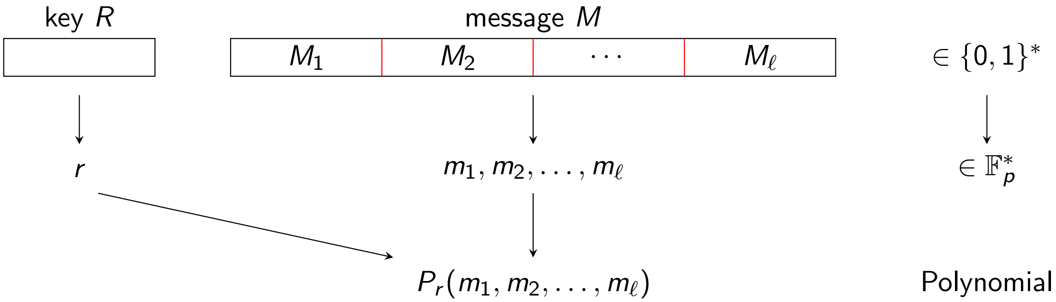
Current Standpoint:

- Broad design space.
- Multiple interactions between available choices.
- Knowledge spreads across research papers, cryptographic libraries, and developers' blogs.

Our Exposition [DGGP24]:



Brief Description of the Design Space



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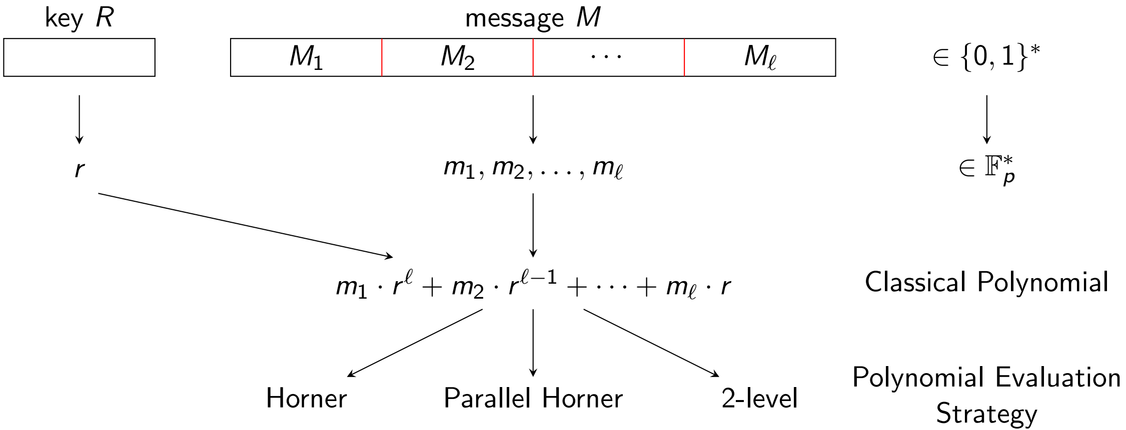

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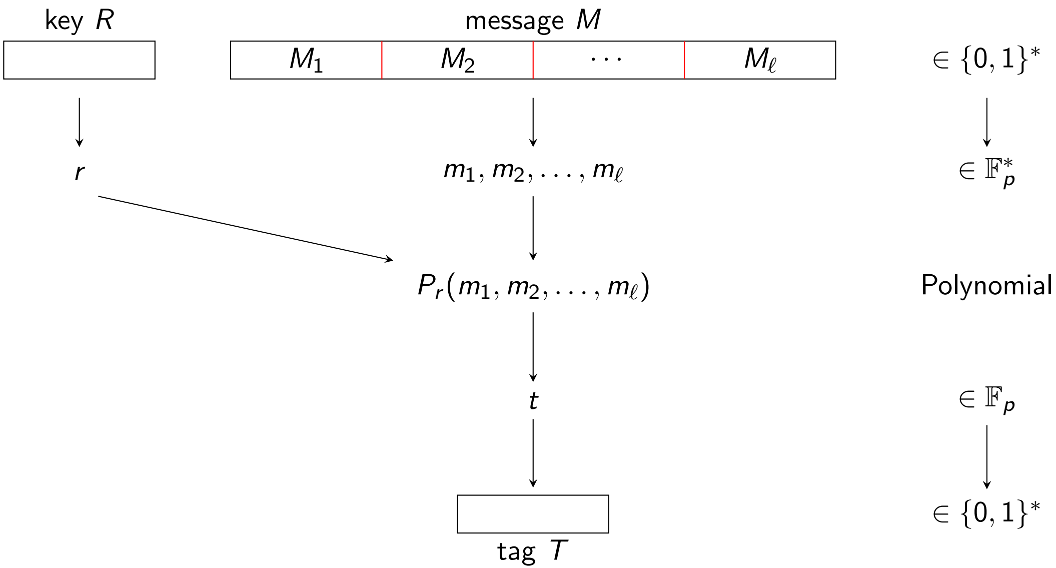
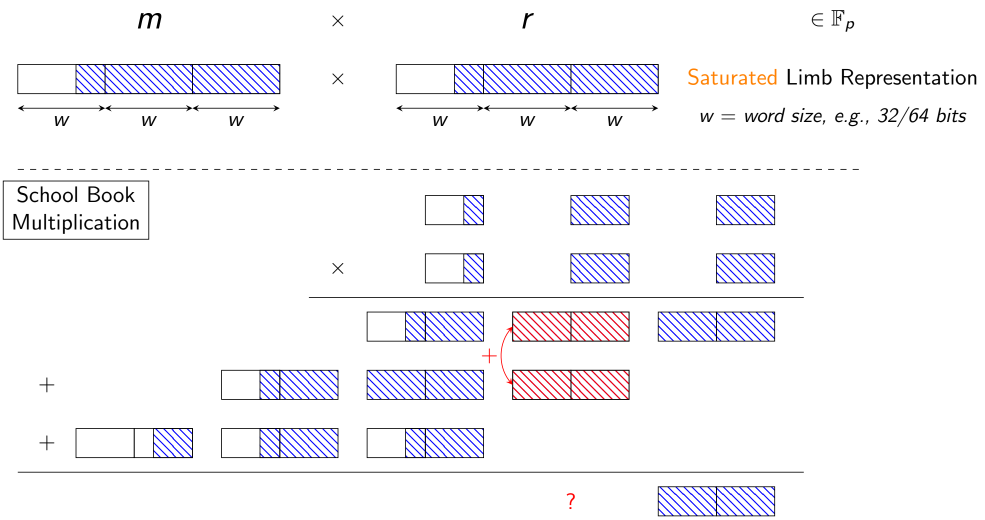
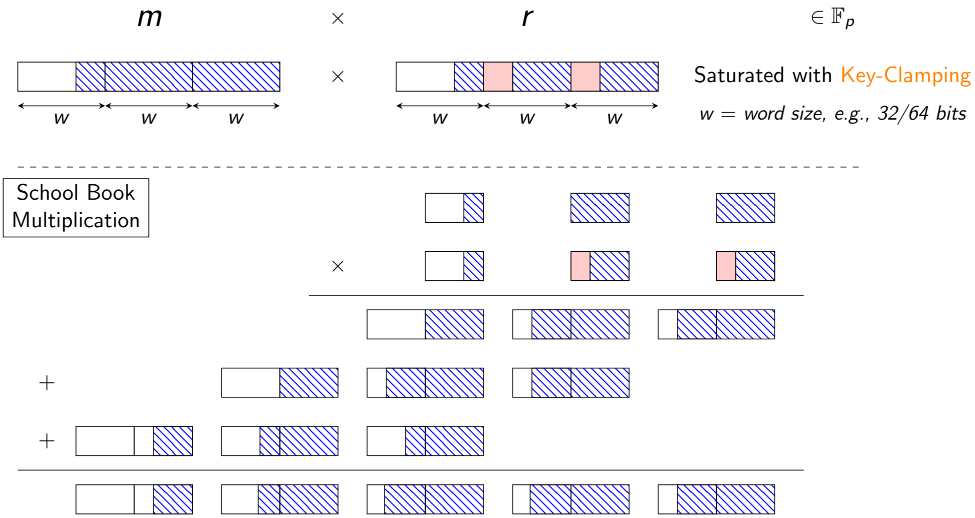


Field Multiplication (Saturated Limb Representation)

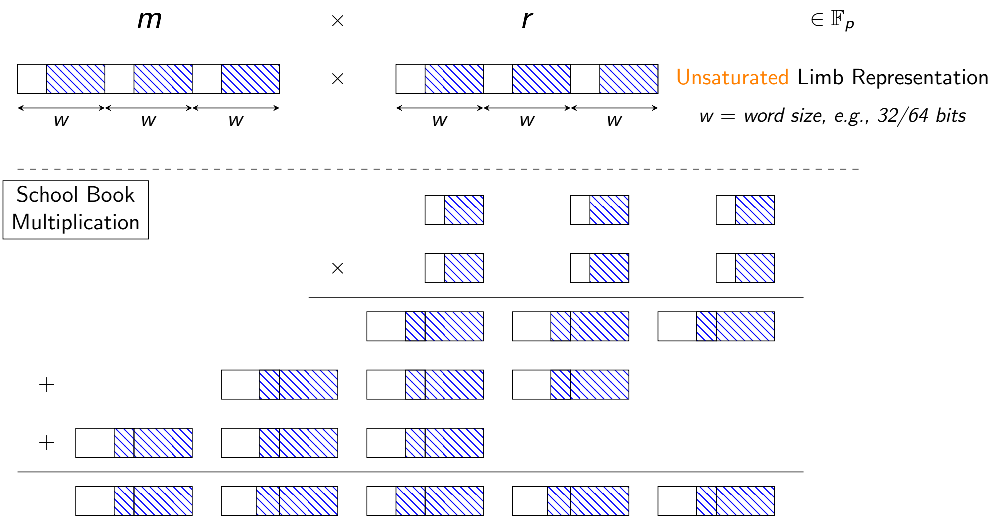


Field Multiplication (Saturated Limb Representation with Key-Clamping)



Limitation: Not exploitable using parallel Horner and 2-level evaluation algorithms.

Field Multiplication (Unsaturated Limb Representation)



Exploitable using parallel Horner and 2-level evaluation algorithms.

Huge Design Space – What Now?

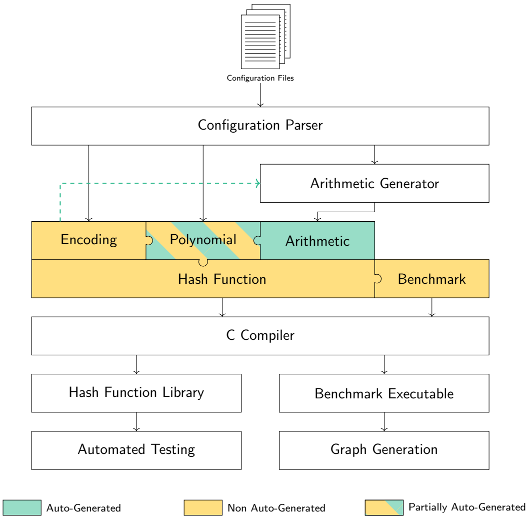
Problem:

- How do we pick a concrete design from this huge space?
- We want to be able to understand and test different combinations.
- Different choices make sense for different hardware.

Solution:

- Modularize!
  - We use our systematization to define modular *configurations*.
- Generic Implementations and Auto-Generation!
  - Write generic implementations, setting specific parameters at compile time.
  - However, fully generic code can lead to bad performance.
  - Where this is likely to occur we automatically generate efficient implementations.

Modular Benchmarking Framework



Goals for New Designs

- More efficient than Poly1305 (i.e., better runtime-security tradeoff).
- Keep things simple, familiar to developers.
- Allow various optimization strategies to tune implementations to different hardware.
- But without tailoring the design towards a specific implementation.
  - Don't design for FPUs!

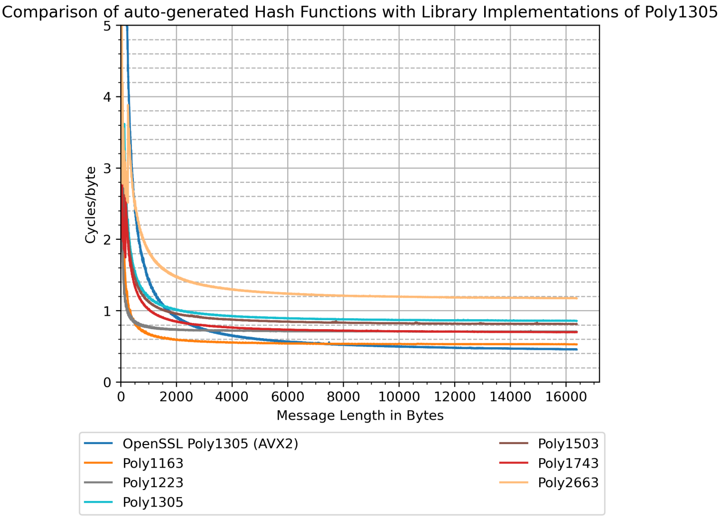
New Designs

- No clamping to support FPU implementations as these are not worth the security loss.
- Stick with Classical Polynomial over $\mathbb{F}_p$. Pack limbs as full as we can.
- Designs allow:
  - Delayed reduction.
  - 2-level polynomial evaluation.
  - Exploiting CPU parallelism.
- 5 designs targeting 3 security-performance tradeoff levels.
  - High Performance at Poly1305 Security.
  - Higher Security at Poly1305 Performance.
  - Very High Security.

New Designs

Design Target	Prime	Bits per limb (32-/64-bit)	Security Level	Hash Function
High Performance at Poly1305 Security	$p_1 = 2^{116} - 3$	29/58	≈107 bits	**Poly1163**
	$p_2 = 2^{122} - 3$	25/61	≈117 bits	**Poly1223**
Higher Security at Poly1305 Performance	$p_3 = 2^{150} - 3$	30/50	≈137 bits	**Poly1503**
	$p_4 = 2^{174} - 3$	29/58	≈161 bits	**Poly1743**
Very High Security	$p_5 = 2^{266} - 3$	27/54	≈245 bits	**Poly2663**

Benchmarking



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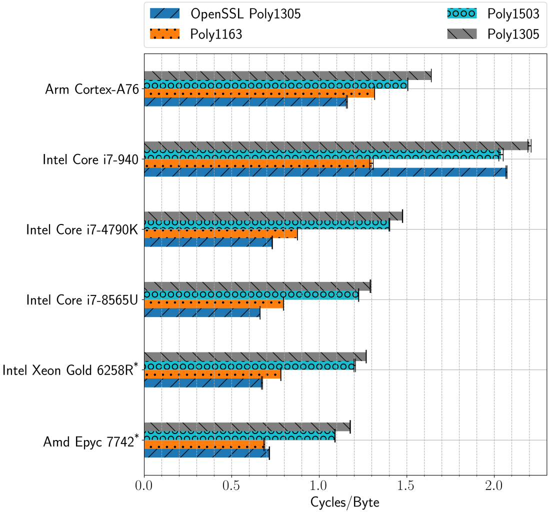

*Turbo Boost/Core Adjusted

Results:

- Our modular implementations achieve **high performance without vectorization or hand-optimization.**
- Poly1163 performance makes it **suitable as drop-in replacement for Poly1305.**

Our Expectations for Vectorization:

- Poly1163: Significantly outperforms Poly1305 at the same security level.
- Poly1503: Replacement for Poly1305 with 34 bits of extra security ($103 \rightarrow 137$) at similar performance.

Where to Find More Details

SoK on Polynomial Hash:

https://doi.ieeecomputersociety.org/10.1109/SP54263.2024.00132

Code of Polynomial Hash Framework:

https://github.com/jangilcher/polynomial_hashing_framework

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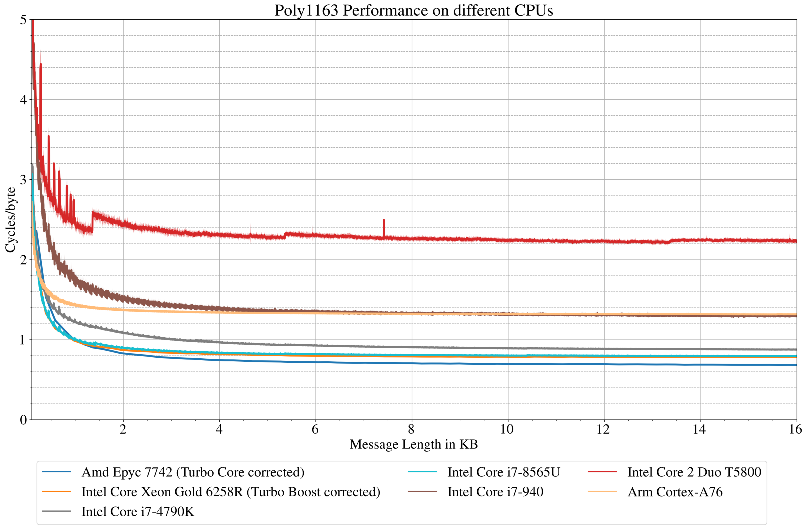
Sok: Efficient design and implementation of polynomial hash functions over prime fields.
In *2024 IEEE Symposium on Security and Privacy (SP)*, pages 131–131, Los Alamitos, CA, USA, may 2024. IEEE Computer Society.

Benchmarks: Poly1163



Benchmarks: Poly1503